Satellite clock steering based on a Kalman Filter

Léo Sol
DTN/TPI/STR
French Space Agency, CNES
Toulouse, France
leo.sol@cnes.fr

David Valat
DTN/TPI/STR
French Space Agency, CNES
Toulouse, France
david.valat@cnes.fr

Yoan Grégoire
DTN/TPI/STR
French Space Agency , CNES
Toulouse, France
yoan.gregoire@cnes.fr

***Abstract*— Due to mechanical and launch constraints, the size of space instruments is limited. A viable solution to circumvent these limitations is the distribution of instruments across a swarm of satellites. This concept of distributed instrument already exists on the ground, as demonstrated by the radio telescope network LOFAR and EHT. To ensure that a swarm of satellites can contribute to a common measurement, each unitary measure needs to be performed in a common timescale. The synchronization performance between the clocks can become particularly crucial, depending on the specific application. Recognizing this, CNES has developed a dedicated test bed to assess the performances of various clocks, identified as prospective candidates for future swarm missions. Furthermore, this test bed serves as a platform for evaluating different clock steering methodologies. This study presents the outcomes of our investigation into a comparison between the classical steering approach and a Kalman filter-based approach, both through simulation and experimental assessments. The outcomes reveal distinct advantages and limitations of each approach: the classical method offers simplicity and ease of implementation, while the Kalman filter-based approach provides robustness to the dynamic space environment, especially in the context of swarms of satellites, where continuous and direct communication between satellites is not guaranteed. The comparative analysis highlights the potential of the Kalman filter-based method as a more robust solution for clock synchronization in satellite swarms.**

Keywords— Satellite swarms, Clock Synchronization, Kalman filter

I. INTRODUCTION

To keep up on driving the space industry innovation which is moving towards new space solutions, French space agency CNES defined ambitious technological development concerning a distribution of instruments across several satellites, referenced to as satellite swarm. Since each instrument needs to perform measurements in a common timescale across the swarm, each unit needs to be synchronized to a common reference clock and it becomes necessary to steer onboard clocks of each unit of the swarm over a reference clock.

In the perspective of missions outside the vicinity of the Earth where GNSS signals aren't available to provide a common time reference, each unit of the swarm must be able to communicate directly one with each other using an inter satellite link (ISL) and estimate the clock offset through signal exchanged between pairs of satellites. However, the ISL adds an additive noise over the measurement of clock offset measurements which needs to be filtered. The Kalman filter is commonly used in space industry and specifically in flight dynamics to perform tracking and steering of positioning parameters and clock frequency.

Furthermore, the communication strategy in a swarm of satellites is a topic under study. For swarms with a large number of satellites, it may be difficult to establish continuous communication between each pair of satellites simultaneously. As a starting point, a time division multiple access (TDMA) method can be considered. In such a communication scheme, clock offset measurements are obtained sequentially. This property will define the context in which the Kalman filter-based steering method will be used and the formalism as well as the configuration of the filter needs to be adapted to that purpose.

II. USE CASE

As briefly mentioned in the introduction part, the clock offset measurements are supposed to be available sequentially and the Kalman filter implementation has to adapted accordingly.

Supposing a communication protocol with a main unit (star topology) that carries a clock with a better stability than the other units, each unit needs to communicate with the main one to steer their onboard clock. To optimize the steering stability, each unit must know their phase difference with the main unit clock as often as possible, meaning the chosen protocol is to have each unit communicate one after the other with the main unit. Preliminary studies in CNES have considered swarms with up to 50 satellites. This number will be considered as one of the assumptions for the current study. Assuming that a phase difference can be estimated every second (this assumption is in line with the capability of some ISL equipment), then a full cycle of measurements has a duration of 50 seconds. In other words, every subordinate unit is able to get an estimation of its phase difference with the main unit once every 50 seconds.

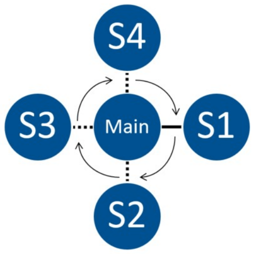

Fig.1. Synoptic diagram of communication protocol

Supposing a perfect communication link between a subordinate unit with the main unit, it would be possible to steer the subordinate unit clock frequency and reach the main's clock stability. To achieve an optimal result, the steering rate depends on the subordinate clock properties and on the measurement noise level. If the steering is performed at a low rate, the subordinate clock will not beneficiate for the main unit clock performance. The sampling time value depends of the considered clock properties and the measurement noise level. To avoid degrading clock short-term stability while benefiting from the main unit clock performance, an optimal sampling time value would be found at 250s considering the assumptions in Fig.2.

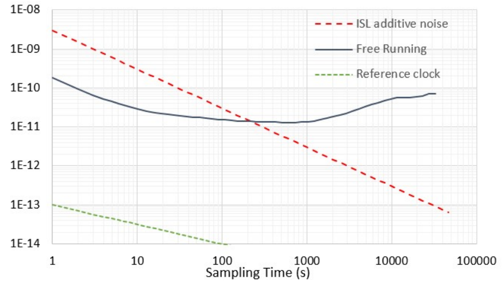

Fig.2. Noise contribution of the ISL compared to the clock to control and the reference clock in a log-log Allan Deviation plot

To elaborate on the short-term steering, using a single noisy clock phase difference measurement to update Kalman filter estimation won't give satisfying results since the uncertainty over the measurement is at the scale of the nanosecond. A better option is to consider at least two measurements for each communication sequence and create a "pre-filter" that uses a linear regression to have a better estimation of the clock difference between two sequences. However, the consequence of using a linear regression considering several measurements is to increase the duration of a time step between two communication sequences of the same unit with the main unit: since a linear regression needs at least 2 measurements, two estimations of the phase difference of the same unit to the main unit of a swarm of 50 subordinate units would be separated by an idle time of 100 s. The steering sampling time is defined as:

$$\tau = N_c * dts$$

With N_c being the number of subordinate units and dts the duration of a communication sequence between the main and the subordinate units. Knowing the noise contribution of the ISL and the type of clock used in the preferred case, an optimal steering sampling time is found at 250 s. Supposing the number of subordinate slave units is set to 50, the optimal duration of a communication sequence is then 5 s. Each subordinate satellite will communicate during 5 s with the main satellite of the swarm cyclically.

Due to this use case, the Kalman filter parameters and formalism [1][2] must be adapted to take into account the number of measurements considered between two iterations of the filter and the type of corrections that are applied to the clock.

III. Formalism

A. Kalman filter formalism

a) Prediction: The prediction step is considering a linear evolution of the estimates over a defined sampling time being the time elapsed between two iterations of the filter. It involves an evolution model matrix $\boldsymbol{F}$ to propagate the estimation made during the last iteration *k-1*. It also involves the current correction applied to the signal $\mathbf{u}_{k-1}$ and the control model $\boldsymbol{B}$ that defines on which estimates the correction applies. The covariance matrix of the estimates predicted is also calculated at the same step. It involves the $\boldsymbol{Q}$ parameter defined as the clock noise model covariance matrix which will be detailed later on.

$$\boldsymbol{F} = \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} \tau \\ 1 \end{bmatrix}$$

$$\boldsymbol{x}_{k|k-1} = \boldsymbol{F}\boldsymbol{x}_{k-1|k-1} + \boldsymbol{B}\boldsymbol{u}_{k-1}$$

$$\boldsymbol{P}_{k|k-1} = \boldsymbol{F}\boldsymbol{P}_{k-1|k-1}\boldsymbol{F}^T + \boldsymbol{Q}$$

b) Measurement: The measurement step isn't part of the classical Kalman filter formalism but is needed in this case. It defines how measurements during a communication sequence are treated and applied to Kalman filtering. As described in II., measurements of phase difference received as input of the Kalman filter are pre-filtered using a linear regression that returns two parameters :

$$z_\varphi = \varphi_k$$

$$z_f = (\varphi_k - \varphi_{k-1})/\tau$$

$$z_k = \begin{bmatrix} z_\varphi \\ z_f \end{bmatrix}$$

φ_k is defined as the phase difference estimated at the end of the linear regression i.e. at the timestamp of the Kalman filter iteration. The measurement vector $\mathbf{z}_k$ will be used as the measurement input to update the Kalman filter.

c) Update: The update step uses the measurement input and compares it with the prediction step output to make an estimation of the phase difference and the frequency difference between subordinate unit clock and main unit clock depending on the trust balance between measurements and clock noise model. The innovation term $\tilde{\mathbf{z}}_k$ contains the residuals between estimates from the prediction step and the measurement input and will be used to calculate the overall estimates of the filter. This estimation is made by comparing the prediction output with the residuals using a gain matrix $\mathbf{K}$. The gain matrix calculation is made using the predicted covariance matrix of the estimates as well as the measurement noise model covariance matrix $\mathbf{R}$. The observation model matrix $\mathbf{H}$ defines how the measurement inputs relates to the estimates:

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\tilde{\boldsymbol{z}}_k = \boldsymbol{x}_{k|k-1} + \boldsymbol{H}\boldsymbol{z}_k$$

$$\boldsymbol{S}_k = \boldsymbol{H}\boldsymbol{P}_{k|k-1}\boldsymbol{H}^T + \boldsymbol{R}$$

$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1}\boldsymbol{H}^T\boldsymbol{S}_k^{-1}$$

$$\boldsymbol{x}_{k|k} = \boldsymbol{x}_{k|k-1} + \boldsymbol{K}_k * \tilde{\boldsymbol{z}}_k$$

$$\boldsymbol{P}_{k|k} = (\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H})\boldsymbol{P}_{k|k-1}$$

B. Modelling the clock noise contribution

Another essential parameter to optimize Kalman filtering is the clock noise model covariance matrix, known as *Q*. Supposing a state vector with two parameters this matrix has a 2x2 size and each term depends of the clock noise model.

Several clock noise models have been calculated in literature and it is a choice made by the user to find the model that describes as best as possible the clock use [3]. In the case of a nanosatellite-class OCXO preferred for satellite swarm missions, the corresponding model is:

$$\sigma_Q^2 = \sigma_{WFM}^2/\tau + \sigma_{RWF}^2 * \tau$$

This model is defined as the sum of the clock white frequency modulation noise σ^2_{WFM} and the clock random walk frequency noise σ^2_{RWF} contributions. Each variance term depends on the clock noise contribution that can be determined by calculating its Allan Deviation (ADEV). By applying a linear regression over the parts of the log-log ADEV representation where the contribution of those terms are dominant over the other terms (respectively -1/2 and +1/2), an estimation of the variance can be made and the model can be calculated [4].

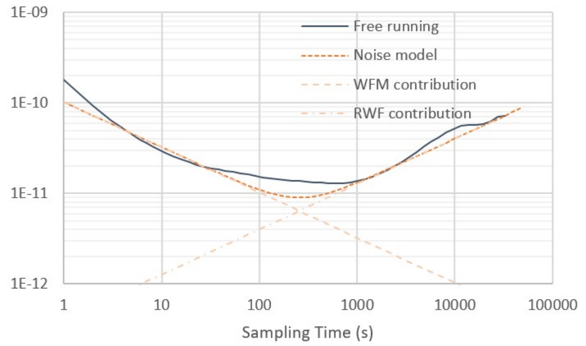

Fig.3. Clock noise model and components

Knowing the noise contribution of the two types of noise mentioned, the covariance matrix of this model can be calculated :

$$Q = \begin{bmatrix} \sigma_{WFM}^2\tau + \sigma_{RWF}^2\tau^3/3 & \sigma_{RWF}^2\tau^2/2 \\ \sigma_{RWF}^2\tau^2/2 & \sigma_{RWF}^2\tau \end{bmatrix}$$

C. Modelling the measurement noise contribution

The last parameter to consider when using the Kalman filter is the measurement noise model covariance matrix noted as ***R***. This matrix is used to provide to the filter an estimation of the uncertainty on the measurements that are used. In a case where we don't know the amplitude of the noise introduced by the equipment, ***R*** can be adjusted dynamically using the innovation term of the Kalman filter (residuals between measurements and prediction). In the current case, additive noise introduced by onboard equipment comes from ISL equipment and can be estimated in advance. The current estimations consider an additive white phase noise with an ADEV of 3E-9 at 1 s.

Due to the use of a linear regression pre-filter, the uncertainty over the measurements decrease since the estimation made by the regression reduces the white noise from the measurements. The standard deviation of the phase difference is then:

$$\sigma_\varphi = \sigma_{ISL}/\sqrt{dts}$$

The use of the linear regression also introduces an estimation of the frequency difference i.e. the slope estimated from the regression. Considering the frequency measurement definition and propagating its uncertainty:

$$\sigma_f = \sigma_{ISL} * \sqrt{2/\tau}$$

The measurement noise model covariance matrix is then defined as follows:

$$R = \begin{bmatrix} \sigma_\varphi^2 & 0 \\ 0 & \sigma_f^2 \end{bmatrix}$$

D. Command law

The command law is defined in the Kalman filter formalism as the correction applied to control the signal depending on the estimations made. The prediction step takes the correction into account so the predicted state represents the steered signal. [5]

Considered clocks can physically be steered using phase and frequency correction. However, the phase correction doesn't have the required precision to match command laws resolution, meaning that each command law is a frequency correction which value depends of the phase and frequency estimation made by the filter.

The command law chosen is the sum of two distinct corrections referred to as phase-locked loop (PLL) and frequency-locked loop (FLL). They have the purpose to respectively control the phase difference and the frequency difference between the two clocks. The FLL is not optimal on its own to minimize the phase difference between the clocks but ensures that the steered clock remains as stable as possible. We define below the two command laws as well the final command law:

$$u_k^{PLL} = -x_{k|k}(1)/\tau$$

$$u_k^{FLL} = -x_{k|k}(2)$$

$$u_k = u_k^{PLL} + u_k^{FLL}$$

The phase difference correction u_k^{PLL} is applied over the entire sequence to minimize its effect over clock frequency stability and also to remain below the maximum frequency steer value allowed by the manufacturer.

IV. Applications

A. Simulation

The software used to perform simulations has been developed to consider several use cases and strategies of communication and taking into account the size of the swarm i.e. the number of satellites. The results shown are calculated using an input data file containing the phase difference between a nanosatellite-class OCXO clock and UTC(CNES) as a reference clock. A second input is needed to simulate the ISL white phase noise considered as an additive noise over the clock differences. Simulations allows verifying the impact on the steering efficiency of various parameters such as the number of satellites and the duration of the communication sequence. The results obtained with the Kalman filter approach will be compared with the "pre-filter" used as a steering command, considered as a reference case.

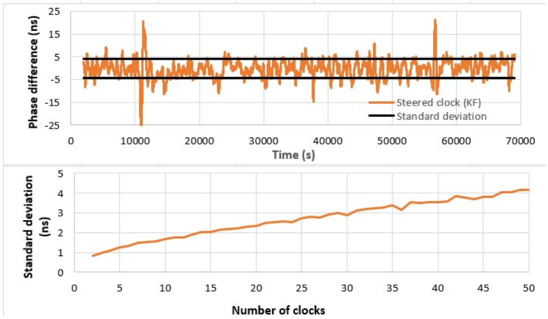

Fig.4. Above : Phase difference between the steered clock and the reference clock for a Nc=50 subordinate units. Below : Internal clock phase standard deviation to the reference clock depending of the number of subordinate units in the swarm.

Figure 4 shows that increasing the number of subordinate units in the swarm also increases the standard deviation of the phase difference. This result is expected since the sampling time between two iterations is extended.

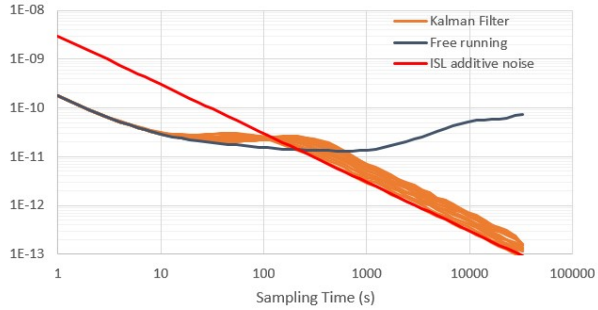

Fig.5. Internal clock ADEV depending of the number of subordinate units in the swarm (from 10 to 50 units)

Another result that shows gains from the use of Kalman filter is shown in the Figure 5. The mention of "internal clock" implies that the clock is measured onboard and doesn't contain measurement noise, even if the measurements for Kalman estimation does. The Kalman filter method filters the short-term additive noise whatever the size of the satellite swarm. The same kind of results has also been found in other use cases [6].

B. Test bench

The CNES Time-Frequency Lab has developed a test bench to test the real-time steering of clocks and to compare the results with simulations.

The test bench compares the output PPS of up to 15 nanosatellite-class OCXO to a stable reference clock being UTC(CNES). Comparisons of PPS outputs are made using a Time Interval Counter and phase difference data are sent to a processing software developed for this study called SWARM_SYNC. An additive white noise is artificially implemented to the raw data to simulate an ISL before sending the data to a second software called SWARM_READER. The purpose of this software is to provide the estimations of the phase and frequency difference between each clock and the reference clock (pre-filtering, Kalman filter) compute the corresponding correction and send it to the timing module.

The size of the swarm can artificially be increased by modifying the sampling time between two measurements of the same clock.

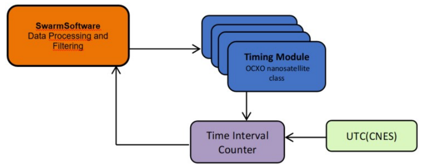

Fig.6. Synoptic diagram of CNES Time-Frequency Lab test bench for clock synchronization

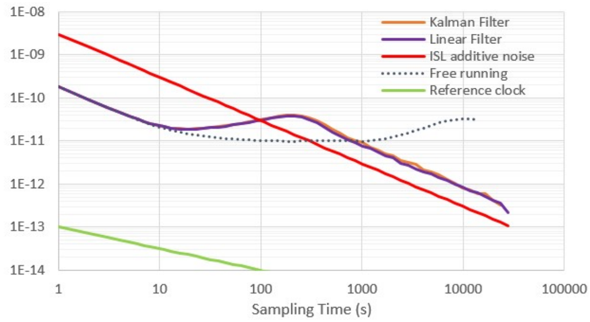

Fig.7. Bench measurements of steered Internal clock ADEV for Nc=50 subordinate units.

V. Discussion

This paper presents simulation and tests results of a synchronization technique based on a Kalman filter, applied on a nanosatellite-class OCXO. Results show different properties of the Kalman filter steering process for satellite swarms.

When a large number of satellites are part of the swarm such as shown in Fig.7, Kalman filter approach does bring the same stability gain that gives the linear filter approach. However, the short-term noise in the Kalman filter approach is filtered whatever the number of satellites in the swarm.

Overall, the synchronization performance provided by Kalman filter steering would be compliant with future swarm mission specifications.

In future works, the study of Kalman filtering robustness to anomalies such as frequency jumps, missing data in communication sequences and holdover mode will be treated. New types of sequences that represents different satellite swarm communication protocols will also be studied to adjust parameters and optimize Kalman filter steering efficiency.

References

[1] R. E. Kalman, "A New Approach to Linear Filtering and Prediction Problems", Journal of Basic Engineering, vol. 82, pp. 35-45, 1960.

[2] L. Galleani and P. Tavella, "Time and the Kalman Filter," IEEE control systems magazine, vol. 30, pp. 44–65, April 2010.

[3] C. Zucca and P. Tavella, "The Clock Model and Its Relationship with the Allan and Related Variances", IEEE Trans. Ultrason., Ferroelect., Freq. Contr., vol. 52, no.2, Feb 2005.

[4] F. Vernotte, E. Lantz, J. Groslambert and J.J. Gagnepain, "Oscillator Noise Analysis: Mutlivariance Measurement", IEEE Trans. Instrum. Meas., vol. 42, no.2, April 1993.

[5] M. Farina, L. Galleani, P. Tavella and S. Bittanti, "A Control Theory Approach to Clock Steering Techniques", IEEE Trans. Ultrason., Ferroelect., Freq. Contr., vol. 57, no.10, Feb 2010.

[6] M. Gödel, T. D. Schmidt and J. Furthner, "Comparison between Simulation and Hardware Realization for Different Clock Steering Techniques" Metrologia, vol.56, no.3, May 2019.